PENN MATHEMATICS ACADEMY SUMMER 2024

The Penn Summer Mathematics Academy will introduce students to a variety of topics in university-level mathematics. We will assume only that students have taken high school Algebra II/Trigonometry. For more information, please check Mathematics Academy Official Website.

1. Instructors

Director:

- Yidi Wang
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Instructors:

- Prof. Henry Towsner
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- Prof. Andrew Cooper
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- Prof. Robert Strain
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- Prof. Davi Maximo
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Teaching Assistant

- Yam Felsenstein
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2. Topics

The Mathematics Academy will have an active learning type of classroom atmosphere where the instructor will give a short lecture and then have the students work on a problem in groups.

There will be optional homework problems assigned each day, designed to help students better learn the material that is being presented. Students are encouraged to work on these homework problems and ask questions during recitation to make sure they understand the material presented that day.

2.1. Introduction proofs.

The Mathematics Academy will start with introductions to proofs, which cover logics, proof techniques like proof by contraction, mathematical induction, etc.

2.2. Topics in Graph Theory (Prof. Towsner).

An introduction to some ideas in graph theory, from the Königsberg bridge problem to the four color theorem, with an emphasis on looking for examples and making conjectures.

2.3. **The Euler Characteristic; or How You Know Your Pants Aren't a Shirt (Prof. Cooper).**

The Euler characteristic describes the shape of an object with a single integer. We'll explore which facets of a shape can be detected using its Euler characteristic, as well as attempting to explain how a single number could possibly contain so much information (and how we can extract even more information with just a bit more math). Time permitting, we will teach a computer to compute Euler characteristics for us so we can understand the shapes of datasets.

2.4. **Difference Equations, and Recurrence Relations (Prof. Strain).**

We will study the theory of difference equations and recurrence relations using a series of interesting and useful real world examples, including compound interest, population growth, bacteria growth, Collatz conjecture ($3n+1$ problem) and matlab, mathematical induction, n choose k and pascal's triangle, identically distributed candy, Tower of Hanoi game, Difference and shift operators, first order difference equations, population growth and interest (again), chaos and the logistic map, chebyshev polynomials, second order difference equations, and the fibonacci sequence.

2.5. **Introduction to Linear Algebras and Applications (Prof. Maximo).**

We will study the basic ideas of linear algebra, including eigenvalues, focusing on the interplay between thinking geometrically and algebraically. We will then discuss some modern applications of linear algebra, such as Google's PageRank algorithm.

2.6. **Numbers and equations.** We will start with elementary number theory, including prime numbers, prime factorizations, congruencies and quadratic reciprocity. Then we will introduce some algebraic structures like groups, rings and fields, and the basic ideas of Galois theory.

3. Daily Schedule

Location: David Rittenhouse Lab A5

Week 1	7/8	7/9	7/10	7/11	7/12
9:30–12:00	Wang	Towsner	Towsner	Towsner	Cooper
12:00–1:30	Lunch Break	Lunch Break	Lunch Break	Lunch Break	Lunch Break
1:30–3:30	Wang	Towsner	Towsner	Towsner	Cooper
4:00–5:00	TA session	TA session	TA session	TA session	TA session

Week 2	7/15	7/16	7/17	7/18	7/19
9:30–12:00	Cooper	Cooper	Strain	Strain	Wang
12:00–1:30	Lunch Break	Lunch Break	Lunch Break	Lunch Break	Lunch Break
1:30–3:30	Cooper	Cooper	Strain	Strain	Wang
4:00–5:00	TA session	TA session	TA session	TA session	TA session

Week 3	7/22	7/23	7/24	7/25	7/26
9:30–12:00	Strain	Maximo	Maximo	Wang	Maximo
12:00–1:30	Lunch Break	Lunch Break	Lunch Break	Lunch Break	Lunch Break
1:30–3:30	Strain	Maximo	Maximo	Wang	Maximo
4:00–5:00	TA session	TA session	TA session	TA session	TA session